Closing Mon, Jan 11: 2.1
Closing Wed, Jan 13: 2.2
Closing Fri, Jan 15: 2.3

2.3 Limit Laws and Strategies

Entry Task: Find the following limits by either plugging in values or graphing

(1) $\lim_{x \to 1} 2x$

(2) $\lim_{x \to 2^+} \frac{x}{x-2}$

(3) $\lim_{x \to 2^-} \frac{x}{x-2}$

(4) $\lim_{x \to \infty} \frac{x}{x-2}$

Some Basic Limit Laws:

1. $\lim_{x \to a} c = c$

2. $\lim_{x \to a} x = a$

3. $\lim_{x \to a} [f(x) + g(x)]$
$$= \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

4. $\lim_{x \to a} [f(x) g(x)]$
$$= \lim_{x \to a} f(x) \lim_{x \to a} g(x)$$

5. If $\lim_{x \to a} g(x) \neq 0$, then
$$\lim_{x \to a} \left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$$

Examples:

1. $\lim_{x \to -7} 10 = 10$

2. $\lim_{x \to 14} x = 14$

3. $\lim_{x \to -2} [x + 6] = \lim_{x \to -2} x + \lim_{x \to -2} 6$

4. $\lim_{x \to 5} [2x^2] = \lim_{x \to 5} 2 \lim_{x \to 5} x \lim_{x \to 5} x$

5. $\lim_{x \to 4} \left[\frac{x+2}{x^2}\right] = \frac{\lim_{x \to 4} (x+2)}{\lim_{x \to 4} x^2}$

Limit Flow Chart for Rational Functions:

$$\lim_{x \to a} \left[\frac{f(x)}{g(x)} \right]$$

1. Try plugging in the value.
   If denominator ≠ 0, done!

2. **If denom = 0 & numerator ≠ 0**, the answer is -∞, +∞ or DNE. Examine the sign of the output from each side.

3. **If denom = 0 & numerator = 0**, Use algebraic methods discussed in class to simplify and cancel until one of them is not zero.

For the den = 0, num = 0 case, here is a summary of the strategies discussed in lecture (we did an example of each):

Strategy 1: Factor/Cancel

Strategy 2: Simplify Fractions

Strategy 3: Expand/Simplify

Strategy 4: Multiply by Conjugate

Strategy 5: Change Variable

Strategy 6: Compare to other functions (Squeeze Thm)

The den = 0, num = 0 case could be called an **indeterminate form**. When we have an indeterminate form, some method (algebra or otherwise) needs to be used to simplify before we can determine the limit.

Besides $\frac{0}{0}$, some other indeterminate forms include

$$\frac{\infty}{\infty}\ , \infty - \infty\ , 1^{\infty}, 0 \cdot \infty, \infty^{0}$$